Queue-length balance equations in multiclass multiserver queues and their generalizations

Marko A.A. Boon[*] Onno J. Boxma [†‡] Offer Kella[§¶] Masakiyo Miyazawa[‖**]

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Abstract

A classical result for the steady-state queue-length distribution of single-class queueing systems is the following: the distribution of the queue length just before an arrival epoch equals the distribution of the queue length just after a departure epoch. The constraint for this result to be valid is that arrivals, and also service completions, with probability one occur individually, i.e., not in batches.

In the first half of this paper, we show that it is easy to write down somewhat similar balance equations for multidimensional queue-length processes for a large family of multiclass multiserver queues with Poisson arrivals – even when arrivals may occur in batches. We demonstrate the use of these balance equations, in combination with PASTA, by (i) providing very simple derivations of some known results for polling systems, and (ii) obtaining new results for some queueing systems with priorities.

In the second half of the paper, we formally verify those balance equations under a general framework. They are called distributional relationships, and are obtained for any external arrival process and state dependent routing as long as certain stationarity conditions are satisfied and external arrivals and service completions do not simultaneously occur. We also extend the distributional relationships for a non-stationary framework.

Keywords: queue length; steady-state distribution; balance equations; distributional relationship; Palm distribution; non-stationary framework.

[*]Department of Mathematics and Computer Science, Eindhoven University of Technology, the Netherlands. m.a.a.boon@tue.nl

[†]Department of Mathematics and Computer Science, Eindhoven University of Technology, the Netherlands. o.j.boxma@tue.nl

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[§]Department of Statistics, The Hebrew University of Jerusalem, Jerusalem 9190501, Israel. offer.kella@gmail.com

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[‖]Department of Information Sciences, Tokyo University of Science, Noda City, Chiba 278, Japan. miyazawa@rs.tus.ac.jp

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