ANNEALED SCALING FOR A CHARGED POLYMER

F. CARAVENNA, F. DEN HOLLANDER, N. PÉTRÉLIS, AND J. POISAT

Abstract. This paper studies an undirected polymer chain living on the one-dimensional integer lattice and carrying i.i.d. random charges. Each self-intersection of the polymer chain contributes to the interaction Hamiltonian an energy that is equal to the product of the charges of the two monomers that meet. The joint probability distribution for the polymer chain and the charges is given by the Gibbs distribution associated with the interaction Hamiltonian. The focus is on the *annealed free energy* per monomer in the limit as the length of the polymer chain tends to infinity.

We derive a *spectral representation* for the free energy and use this to prove that there is a critical curve in the parameter plane of charge bias versus inverse temperature separating a *ballistic phase* from a *subballistic phase*. We show that the phase transition is *first order*. We prove large deviation principles for the laws of the empirical speed and the empirical charge, and derive a spectral representation for the associated rate functions. Interestingly, in both phases both rate functions exhibit *flat pieces*, which correspond to an inhomogeneous strategy for the polymer to realise a large deviation. The large deviation principles in turn lead to laws of large numbers and central limit theorems. We identify the scaling behaviour of the critical curve for small and for large charge bias. In addition, we identify the scaling behaviour of the free energy for small charge bias and small inverse temperature. Both are linked to an associated *Sturm-Liouville eigenvalue problem.*

A key tool in our analysis is the Ray-Knight formula for the local times of one-dimensional simple random walk. This formula is exploited to derive a *closed form expression* for the generating function of the annealed partition function, and for several related quantities. This expression in turn serves as the starting point for the derivation of the spectral representation for the free energy, and for the scaling theorems.

What happens for the *quenched free energy* per monomer remains open. We state two modest results and raise a few questions.

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