Parabolic Anderson model in a dynamic random environment: random conductances

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July 21, 2015

Abstract

The parabolic Anderson model is defined as the partial differential equation $\partial u(x,t)/\partial t = \kappa\Delta u(x,t) + \xi(x,t)u(x,t)$, $x \in \mathbb{Z}^d$, $t \geq 0$, where $\kappa \in [0,\infty)$ is the diffusion constant, Δ is the discrete Laplacian, and ξ is a *dynamic random environment* that drives the equation. The initial condition $u(x,0) = u_0(x)$, $x \in \mathbb{Z}^d$, is typically taken to be non-negative and bounded. The solution of the parabolic Anderson equation describes the evolution of a field of particles performing independent simple random walks with binary branching: particles jump at rate $2d\kappa$, split into two at rate $\xi \vee 0$, and die at rate $(-\xi) \vee 0$. In earlier work we looked at the *Lyapunov exponents*

$$\lambda_p(\kappa) = \lim_{t\to\infty} \frac{1}{t} \log \mathbb{E}([u(0,t)]^p)^{1/p}, \quad p \in \mathbb{N}, \qquad \lambda_0(\kappa) = \lim_{t\to\infty} \frac{1}{t} \log u(0,t).$$

For the former we derived *quantitative* results on the κ-dependence for four choices of ξ: space-time white noise, independent simple random walks, the exclusion process and the voter model. For the latter we obtained *qualitative* results under certain space-time mixing conditions on ξ.

In the present paper we investigate what happens when $\kappa\Delta$ is replaced by $\Delta^{\mathcal{K}}$, where $\mathcal{K} = \{\mathcal{K}(x,y)\colon\, x,y \in \mathbb{Z}^d,\, x \sim y\}$ is a collection of random conductances between neighbouring sites replacing the constant conductances κ in the homogeneous model. We show that the associated *annealed* Lyapunov exponents $\lambda_p(\mathcal{K})$, $p \in \mathbb{N}$, are given by the formula

$$\lambda_p(\mathcal{K}) = \sup\{\lambda_p(\kappa)\colon\, \kappa \in \mathrm{Supp}(\mathcal{K})\},$$

where $\mathrm{Supp}(\mathcal{K})$ is the set of values taken by the $\mathcal{K}$-field. We also show that for the associated *quenched* Lyapunov exponent $\lambda_0(\mathcal{K})$ this formula only provides a lower bound, and we conjecture that an upper bound holds when $\mathrm{Supp}(\mathcal{K})$ is replaced by its convex hull. Our proof is valid for three classes of reversible ξ, and for all $\mathcal{K}$ satisfying a certain *clustering property*, namely, there are arbitrarily large balls where $\mathcal{K}$ is almost constant and close to any value in $\mathrm{Supp}(\mathcal{K})$. What our result says is that the Lyapunov exponents are controlled by those pockets of $\mathcal{K}$ where the conductances are close to the value that maximises the growth in the homogeneous setting. Our proof is based on variational representations and confinement arguments showing that mixed pockets are subdominant.

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