A quenched large deviation principle in a continuous scenario

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Abstract

We prove the analogue for continuous space-time of the quenched LDP derived in Birkner, Greven and den Hollander [2] for discrete space-time. In particular, we consider a random environment given by Brownian increments, cut into pieces according to an independent continuous-time renewal process. We look at the empirical process obtained by recording both the length of and the increments in the successive pieces. For the case where the renewal time distribution has a Lebesgue density with a polynomial tail, we derive the quenched LDP for the empirical process, i.e., the LDP conditional on a typical environment. The rate function is a sum of two specific relative entropies, one for the pieces and one for the concatenation of the pieces. We also obtain a quenched LDP when the tail decays faster than algebraic. The proof uses coarse-graining and truncation arguments, involving various approximations of specific relative entropies that are not quite standard.

In a companion paper we show how the quenched LDP and the techniques developed in the present paper can be applied to obtain a variational characterisation of the free energy and the phase transition line for the Brownian copolymer near a selective interface.

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