Variational description of Gibbs-non-Gibbs dynamical transitions for spin-flip systems with a Kac-type interaction

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Abstract

We continue our study of Gibbs-non-Gibbs dynamical transitions. In the present paper we consider a system of Ising spins on a large discrete torus with a Kac-type interaction subject to an independent spin-flip dynamics (infinite-temperature Glauber dynamics). We show that, in accordance with the program outlined in [11], in the thermodynamic limit Gibbs-non-Gibbs dynamical transitions are *equivalent* to bifurcations in the set of global minima of the large-deviation rate function for the trajectories of the empirical density *conditional* on their endpoint. More precisely, the time-evolved measure is non-Gibbs if and only if this set is not a singleton for *some* value of the endpoint. A partial description of the possible scenarios of bifurcation is given, leading to a characterization of passages from Gibbs to non-Gibbs and vice versa, with sharp transition times.

Our analysis provides a conceptual step-up from our earlier work on Gibbs-non-Gibbs dynamical transitions for the Curie-Weiss model, where the mean-field interaction allowed us to focus on trajectories of the empirical magnetization rather than the empirical density.

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