Phase diagram for a copolymer in a micro-emulsion

F. den Hollander [1]
N. Pétrélis [2]

September 4, 2013

Abstract

In this paper we study a model describing a copolymer in a micro-emulsion. The copolymer consists of a random concatenation of hydrophobic and hydrophilic monomers, the micro-emulsion consists of large blocks of oil and water arranged in a percolation-type fashion. The interaction Hamiltonian assigns energy $-\alpha$ to hydrophobic monomers in oil and energy $-\beta$ to hydrophilic monomers in water, where α, β are parameters that without loss of generality are taken to lie in the cone $\{(\alpha,\beta) \in \mathbb{R}^2\colon\ \alpha \geq |\beta|\}$. Depending on the values of these parameters, the copolymer either stays close to the oil-water interface (localization) or wanders off into the oil and/or the water (delocalization). We derive two variational formulas for the quenched free energy per monomer, one that is "column-based" and one that is "slope-based". Using these variational formulas we identify the phase diagram in the (α,β)-cone. There are two regimes: *supercritical* (the oil blocks percolate) and *subcritical* (the oil blocks do not percolate). The supercritical and the subcritical phase diagram each have two localized phases and two delocalized phases, separated by four critical curves meeting at a quadruple critical point. The different phases correspond to the different ways in which the copolymer can move through the micro-emulsion.

AMS 2000 *subject classifications.* 60F10, 60K37, 82B27.
Key words and phrases. Random copolymer, random micro-emulsion, free energy, percolation, variational formula, large deviations, concentration of measure.
Acknowledgment: The research in this paper is supported by ERC Advanced Grant 267356 VARIS. NP is grateful for hospitality at the Mathematical Institute of Leiden University during extended visits in 2011, 2012 and 2013.
Remark: The part of this paper dealing with the "column-based" variational formula for the free energy has appeared as a preprint on the mathematics archive: arXiv:1204.1234.

[1] Mathematical Institute, Leiden University, P.O. Box 9512, 2300 RA Leiden, The Netherlands
[2] Laboratoire de Mathématiques Jean Leray UMR 6629, Université de Nantes, 2 Rue de la Houssinière, BP 92208, F-44322 Nantes Cedex 03, France