Extremal geometry of a Brownian porous medium

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Abstract

The path $W[0,t]$ of a Brownian motion on a d-dimensional torus $\mathbb{T}^d$ run for time t is a random compact subset of $\mathbb{T}^d$. We study the geometric properties of the complement $\mathbb{T}^d \backslash W[0,t]$ as $t \to \infty$ for $d \geq 3$. In particular, we show that the largest region in this complement has linear scale $\varphi_d(t) = [(d \log t)/(d-2)\kappa_d t]^{1/(d-2)}$, where κ_d is the capacity of the unit ball. More specifically, we identify the sets E for which $\mathbb{T}^d \setminus W[0,t]$ contains a translate of $\varphi_d(t)E$, and we count the number of such translates. Furthermore, we derive large deviation principles for the largest inradius as $t \to \infty$ and the ϵ-cover time as $\epsilon \downarrow 0$. Our results, which generalise laws of large numbers proved in [9], are based on a large deviation principle for the shape of the component with largest capacity in $\mathbb{T}^d \setminus W_{\rho(t)}[0,t]$, where $W_{\rho(t)}[0,t]$ is the Wiener sausage of radius $\rho(t)$ chosen such that $\varphi_d(t)/(\log t)^{1/d} \ll \rho(t) \ll \varphi_d(t)$ as $t \to \infty$. The idea behind this choice is that $\mathbb{T}^d \backslash W[0,t]$ consists of "lakes" whose linear size is of order $\varphi_d(t)$, connected by narrow "channels" whose linear size is of order at most $\varphi_d(t)/(\log t)^{1/d}$. We also derive large deviation principles for the principal Dirichlet eigenvalue and for the maximal volume of the components of $\mathbb{T}^d \setminus W_{\rho(t)}[0,t]$ as $t \to \infty$.

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