Kawasaki dynamics with two types of particles: critical droplets

F. den Hollander [1] [2]
F.R. Nardi [3] [2]
A. Troiani [1]

July 31, 2012

Abstract

This is the third in a series of three papers in which we study a two-dimensional lattice gas consisting of two types of particles subject to Kawasaki dynamics at low temperature in a large finite box with an open boundary. Each pair of particles occupying neighboring sites has a negative binding energy provided their types are different, while each particle has a positive activation energy that depends on its type. There is no binding energy between particles of the same type. At the boundary of the box particles are created and annihilated in a way that represents the presence of an infinite gas reservoir. We start the dynamics from the empty box and are interested in the transition time to the full box. This transition is triggered by a critical droplet appearing somewhere in the box.

In the first paper we identified the parameter range for which the system is metastable, showed that the first entrance distribution on the set of critical droplets is uniform, computed the expected transition time up to and including a multiplicative factor of order one, and proved that the nucleation time divided by its expectation is exponentially distributed, all in the limit of low temperature. These results were proved under *three hypotheses*, and involved *three model-dependent quantities*: the energy, the shape and the number of critical droplets. In the second paper we proved the first and the second hypothesis and identified the energy of critical droplets. In the third paper we prove the third hypothesis and identify the shape and the number of critical droplets, thereby completing our analysis.

Both the second and the third paper deal with understanding the *geometric properties* of subcritical, critical and supercritical droplets, which are crucial in determining the metastable behavior of the system, as explained in the first paper. The geometry turns out to be considerably more complex than for Kawasaki dynamics with one type of particle, for which an extensive literature exists. The main motivation behind our work is to understand metastability of multi-type particle systems.

MSC2010. 60K35, 82C20, 82C22, 82C26, 05B50.
Key words and phrases. Multi-type lattice gas, Kawasaki dynamics, metastability, critical droplets, polyominoes, discrete isoperimetric inequalities.

[1] Mathematical Institute, Leiden University, P.O. Box 9512, 2300 RA Leiden, The Netherlands
[2] EURANDOM, P.O. Box 513, 5600 MB Eindhoven, The Netherlands
[3] Technische Universiteit Eindhoven, P.O. Box 513, 5600 MB Eindhoven, The Netherlands