Free energy of a copolymer in a micro-emulsion

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Abstract

In this paper we consider a two-dimensional model of a copolymer consisting of a random concatenation of hydrophilic and hydrophobic monomers, immersed in a micro-emulsion of random droplets of oil and water. The copolymer interacts with the micro-emulsion through an interaction Hamiltonian that favors matches and disfavors mismatches between the monomers and the solvents, in such a way that the interaction with the oil is stronger than with the water.

The configurations of the copolymers are directed self-avoiding paths in which only steps up, down and right are allowed. The configurations of the micro-emulsion are square blocks with oil and water arranged in percolation-type fashion. The only restriction imposed on the path is that in every column of blocks its vertical displacement on the block scale is bounded. The way in which the copolymer enters and exits successive columns of blocks is a directed self-avoiding path as well, but on the block scale. We refer to this path as the coarse-grained self-avoiding path. We are interested in the limit as the copolymer and the blocks become large, in such a way that the copolymer spends a long time in each block yet visits many blocks. This is a coarse-graining limit in which the space-time scales of the copolymer and of the micro-emulsion become separated.

We derive a *variational formula* for the *quenched free energy per monomer*, where quenched means that the disorder in the copolymer and the disorder in the micro-emulsion are both frozen. In a sequel paper we will analyze this variational formula and identify the phase diagram. It turns out that there are two regimes, *supercritical* and *subcritical*, depending on whether the oil blocks percolate or not along the coarse-grained self-avoiding path. The phase diagrams in the two regimes turn out to be completely different.

In earlier work we considered the same model, but with an unphysical restriction: paths could enter and exit blocks only at diagonally opposite corners. Without this restriction, the variational formula for the quenched free energy is more complicated, but in the sequel paper we will see that it is still tractable enough to allow for a qualitative analysis of the phase diagram.

Part of our motivation is that our model can be viewed as a coarse-grained version of the well-known *directed polymer with bulk disorder*. The latter has been studied intensively in the literature, but no variational formula is as yet available.

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