Variational description of Gibbs-non-Gibbs dynamical transitions for the Curie-Weiss model

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February 2, 2012

Abstract

We perform a detailed study of Gibbs-non-Gibbs transitions for the Curie-Weiss model subject to independent spin-flip dynamics ("infinite-temperature" dynamics). We show that, in this setup, the program outlined in van Enter, Fernández, den Hollander and Redig can be fully completed, namely, we show that Gibbs-non-Gibbs transitions are *equivalent* to bifurcations in the set of global minima of the large-deviation rate function for the trajectories of the magnetization *conditioned* on their endpoint. As a consequence, we show that the time-evolved model is non-Gibbs if and only if this set is not a singleton for *some* value of the final magnetization. A detailed description of the possible scenarios of bifurcation is given, leading to a full characterization of passages from Gibbs to non-Gibbs and back, with sharp transition times (i.e., under the dynamics Gibbsianness can be lost and can be recovered).

Our analysis expands the work of Ermolaev and Külske, who considered zero magnetic field and finite-temperature spin-flip dynamics. We consider both zero and non-zero magnetic field, but restrict to infinite-temperature spin-flip dynamics. Our results reveal an interesting dependence on the interaction parameters, including the presence of forbidden regions for the optimal trajectories and the possible occurrence of overshoots and undershoots in the optimal trajectories. The numerical plots provided are obtained with the help of MATHEMATICA.

MSC 2010. 60F10, 60K35, 82C22, 82C27.
Key words and phrases. Curie-Weiss model, spin-flip dynamics, Gibbs vs. non-Gibbs, dynamical transition, large deviations, action integral, bifurcation of rate function.
Acknowledgment. FdH is supported by ERC Advanced Grant VARIS-267356. JM is supported by Erasmus Mundus scholarship BAPE-2009-1669. The authors are grateful to A. van Enter, V. Ermolaev, C. Külske, A. Opoku and F. Redig for discussions.

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