A copolymer near a selective interface: variational characterization of the free energy

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Abstract

In this paper we consider a random copolymer near a selective interface separating two solvents. The configurations of the copolymer are directed paths that can make i.i.d. excursions of finite length above and below the interface. The excursion length distribution is assumed to have a tail that is logarithmically equivalent to a power law with exponent $\alpha \geq 1$. The monomers carry i.i.d. real-valued types whose distribution is assumed to have zero mean, unit variance, and a finite moment generating function. The interaction Hamiltonian rewards matches and penalizes mismatches of the monomer types and the solvents, and depends on two parameters: the interaction strength $\beta \geq 0$ and the interaction bias $h \geq 0$. We are interested in the behavior of the copolymer in the limit as its length tends to infinity.

The quenched free energy per monomer $(\beta, h) \mapsto g^{\rm que}(\beta, h)$ has a phase transition along a quenched critical curve $\beta \mapsto h_c^{\rm que}(\beta)$ separating a localized phase, where the copolymer stays close to the interface, from a delocalized phase, where the copolymer wanders away from the interface. We derive *variational formulas* for both these quantities. We compare these variational formulas with their analogues for the annealed free energy per monomer $(\beta, h) \mapsto g^{\rm ann}(\beta, h)$ and the annealed critical curve $\beta \mapsto h_c^{\rm ann}(\beta)$, both of which are explicitly computable. This comparison leads to:

(1) A proof that $h_c^{\rm ann}(\beta/\alpha) < h_c^{\rm que}(\beta) < h_c^{\rm ann}(\beta)$ for all $\alpha > 1$ and $\beta > 0$.

(2) A proof that $g^{\rm que}(\beta, h) < g^{\rm ann}(\beta, h)$ for all $\alpha \geq 1$ and (β, h) in the annealed localized phase.

(3) An estimate of the total number of times the copolymer visits the interface in the interior of the quenched delocalized phase.

(4) An identification of the asymptotic frequency at which the copolymer visits the interface in the quenched localized phase.

The copolymer model has been studied extensively in the literature. The goal of the present paper is to open up a window with a variational view and to settle a number of open problems.

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