

## Metastability for Kawasaki dynamics at low temperature with two types of particles

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**Abstract:** This is the first in a series of three papers in which we study a two-dimensional lattice gas consisting of two types of particles subject to Kawasaki dynamics at low temperature in a large finite box with an open boundary. Each pair of particles occupying neighboring sites has a negative binding energy provided their types are different, while each particle has a positive activation energy that depends on its type. There is no binding energy between neighboring particles of the same type. We start the dynamics from the empty box and compute the transition time to the full box. This transition is triggered by a *critical droplet* appearing somewhere in the box.

We identify the region of parameters for which the system is metastable. For this region, in the limit as the temperature tends to zero, we show that the first entrance distribution on the set of critical droplets is uniform, compute the expected transition time up to and including a multiplicative factor of order one, and prove that the transition time divided by its expectation is exponentially distributed. These results are derived under *three hypotheses*, which are verified in the second and the third paper for a certain subregion of the metastable region. These hypotheses involve three model-dependent quantities – the energy, the shape and the number of the critical droplets – which are identified in the second and the third paper as well.

The main motivation behind this work is to understand metastability of multi-type particle systems. It turns out that for two types of particles the *geometry* of subcritical and critical droplets is more complex than for one type of particle. Consequently, it is a somewhat delicate matter to capture the proper mechanisms behind the growing and the shrinking of subcritical droplets until a critical droplet is formed. Our proofs use potential theory and rely on ideas developed in [?] for Kawasaki dynamics with one type of particle. Our target is to identify the *minimal* hypotheses that are needed for metastable behavior.

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