Quenched Lyapunov exponent for the parabolic Anderson model in a dynamic random environment

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Abstract We continue our study of the parabolic Anderson equation $\partial u/\partial t = \kappa\Delta u + \gamma\xi u$ for the space-time field $u\colon\ \mathbb{Z}^d \times [0,\infty) \to \mathbb{R}$, where $\kappa \in [0,\infty)$ is the diffusion constant, Δ is the discrete Laplacian, $\gamma \in (0,\infty)$ is the coupling constant, and $\xi\colon\ \mathbb{Z}^d \times [0,\infty) \to \mathbb{R}$ is a space-time random environment that drives the equation. The solution of this equation describes the evolution of a "reactant" u under the influence of a "catalyst" ξ, both living on $\mathbb{Z}^d$.

In earlier work we considered three choices for ξ: independent simple random walks, the symmetric exclusion process, and the symmetric voter model, all in equilibrium at a given density. We analyzed the *annealed* Lyapunov exponents, i.e., the exponential growth rates of the successive moments of u w.r.t. ξ, and showed that these exponents display an interesting dependence on the diffusion constant κ, with qualitatively different behavior in different dimensions d. In the present paper we focus on the *quenched* Lyapunov exponent, i.e., the exponential growth rate of u conditional on ξ.

We first prove existence and derive some qualitative properties of the quenched Lyapunov exponent for a general ξ that is stationary and ergodic w.r.t. translations in $\mathbb{Z}^d$ and satisfies certain noisiness conditions. After that we focus on the three particular choices for ξ mentioned above and derive some more detailed properties. We close by formulating a number of open problems.

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