On the work of Andrei Okounkov on random partitions

Andrei Okounkov made substantial contributions to a wide variety of fields: group representation theory, asymptotic combinatorics, ergodic theory, theory of random surfaces, algebraic geometry and mathematical physics. Random partitions is one of the principal instruments from his toolbox and a recurrent theme of his versatile research.

A partition λ of an integer n is a nonincreasing sequence of positive integers whose sum is n. Similarly, a plane partition is a matrix of nonnegative integers with nonincreasing rows and columns. Partitions are very basic objects which appear throughout in mathematics, often as indices or summation ranges. A viewpoint stressed by Okounkov is that such sums can be treated probabilistically, as expectations of functionals of partitions under suitable probability measures.

In this talk we shall briefly survey the appearances of partitions in the work of Okounkov and his collaborators. This includes dimers and random surfaces. Then we shall turn to his earlier work on asymptotics of the Plancherel measure and its generalisations. The Plancherel measure gives to each partition λ the probability $(dim\lambda)^2/n!$ where $dim\lambda$ is the dimension of the corresponding representation of the symmetric group (where $dim\lambda$ is computable e.g. by the hook formula). One of the highlights of Okounkov's work in this direction was the first proof of the Baik-Deift-Johansson conjecture, which stated that the asymptotic distribution of the parts of λ coincides with the asymptotic distribution of ordered eigenvalues of large random Hermitian matrices.