

# Portfolio optimization with translation invariant and positive homogeneous risk measure and minimization of the root of quadratic functional

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## Abstract

There is a wide-spread opinion that for elliptically distributed risks (which also include multivariate normally distributed stock returns) the use of any positive homogeneous and translation invariant risk measure for the optimal portfolio management results in the same solution as does the mean-variance risk measure. This solution is equivalent to a well-known Markowitz portfolio (see, for example, McNeil, Frey and Embrechts (2005), Section 6.1.5, where the problem was well documented). Let us note, however, that such a conclusion is only true, when among the linear constraints on the weights of the portfolio diversification the certainty of the expected portfolio risk is required. If the latter restriction is not included in the set of linear portfolio constraints the solution is distinct from that obtained by mean-variance risk measure. We give the condition when this portfolio management problem as well as the problem of the minimization of the root of quadratic functional has solution and provide with the explicit closed form solution. As a main corollary we solve the problem of minimizing the VaR and expected short fall risk measures. The results are demonstrated with the data of 10 stocks from NASDAQ/Computers.

## References

- [1] McNeil, A.,J., Frey, R., Embrechts, P. (2005). *Quantitative Risk Management*. Princeton University Press, Princeton.