# Some results on some interacting systems

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In the first part of the presentation, we consider the following interacting system

$$dX_t^i = (\sum_{j\in\Lambda_i} A_{ji}X_t^j + U_i(X_t^{\Lambda_i}))dt + dZ_t^i \quad (i \in \mathbb{Z}^d)$$

where $A_{ii} = -1$, $A_{ij} \geq 0$ $(i \neq j)$, $\Lambda_0 \subset\subset \mathbb{Z}^d$ is a finite cube including 0, $\Lambda_i = \{j + i : j \in \Lambda_0\}$, and $\{Z_t^i\}_{i\in\mathbb{Z}}$ are a sequence of i.i.d. $\alpha$-stable processes $(1 < \alpha \leq 2)$. We prove the following ergodic result:

**Theorem:** Set

$$\Omega = \mathbb{R}^{\mathbb{Z}^d},\ |i| = \sum_{k=1}^d |i_k|\ (i \in \mathbb{Z}^d),$$
$$B_{R,\rho} = \{x \in \Omega : x = (x_i)_{i\in\mathbb{Z}^d},\ |x_i| \leq R|i|^\rho\} \quad (R > 0, \rho > 0), \mathbb{B} = \bigcup_{R>0,\rho>0} B_{R,\rho}.$$

If

$$\sup_{i\in\mathbb{Z}^d} \sum_{j\in\Lambda_i} (A_{ji} + ||\partial_j U_i||) < c\ (\textit{a small positive constant}),$$

then for any $C^1$ cylinder function $f : \Omega \to \mathbb{R}$, there exists a constant $a$ such that

$$\lim_{t\to\infty} P_t f(x) = a \qquad \forall\ x \in \mathbb{B}.$$

where $P_t$ is the semigroup generated by the above stochastic system.

In the second part, we consider a mathematical generalization of Glauber dynamics described by the following differential equation:

$$\begin{cases} \partial_t u = \sum_{i\in\mathbb{Z}^d} \{\phi^{-1} E_{\Lambda_i}[\phi(f)] - f\} \\ u(0) = f \end{cases}$$

where $\phi$ is some increasing convex function, which has infinitely many choices. As $\phi(x) = x$, the system turns to be (heat bath) Glauber dynamics. We talk about the large time behaviors of the system such as the ergodic property and about some inequalities between the initial data $f$ and the solution $u$. When $\phi = e^x$, we build the relationship between the nonlinear system and the linear one, and show by the former inequalities that the Gibbs measure is the tangent functional of some nonlinear function generated by the system.