**A minicourse on first-passage percolation**

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First-passage percolation is a probabilistic model for the spread of a disease. In the simplest version one assigns a random passage time $t(e)$ to each edge $e$ of $\mathbb{Z}^d$. The random variables $\{t(e) : e \in \mathbb{Z}^d\}$ are assumed independent and nonnegative, all with the same distribution function $F$. For two sets $A, B \subset \mathbb{Z}^d$ the passage time $T(A,B)$ is defined as

$$T(A,B) = \inf \Big\{ \sum_{i=1}^{n} t(e_i) : (e_1, e_2, \dots, e_n) \text{ is a path from } A \text{ to } B \Big\}.$$

The main object of study is the set $B(t) := \big\{ v \in \mathbb{Z}^d : T\{\{\mathbf{0}\}, \{v\} \le t \big\}$. This is the set of points which can be reached from the origin by time $t$. The "shape theorem" is a kind of strong law of large numbers for $B(t)$. It says that under mild conditions $t^{-1}B(t)$ converges almost surely to a non random set $B_0$. There is much interest these days in the fluctations of the boundary of $B(t)$ (which corresponds to a distributional limit theorem beyond the strong law of large numbers).

We shall discuss the basic properties of this model and "prove" the shape theorem and some of the results known so far on the fluctuations of the boundary of $B(t)$.

If time permits we will also discuss a similar process which we call last-passage percolation. This considers a supremum instead of the inf in $T(A,B)$. More specifically, one considers

$$N_n = \sup \Big\{ \sum_{i=1}^{n} t(e_i) : (e_1, e_2, \dots, e_n) \text{ is a connected set of } n \text{ edges,} \\ \text{containing the origin} \Big\}$$

and related suprema. Now one is interested in the asymptotic behavior of and limit laws for $N_n$ as $n \to \infty$.