Diffusion in an annihilating environment

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Abstract

This is joint work with F. den Hollander and S. Molchanov. We study the following reaction-diffusion problem:

$$\begin{aligned} \partial\varrho/\partial t &= \Delta\varrho - V\varrho + \lambda\delta_0, & \varrho(0,x) &\equiv 0, \\ \partial V/\partial t &= -\varrho V, & V(0,x) &\equiv 1. \end{aligned}$$

This system describes a continuum version of a model in which particles are injected at the origin at rate λ, perform independent simple symmetric random walks on $\mathbb{Z}^d$, and are annihilated at rate 1 by traps located at the sites of $\mathbb{Z}^d$ in such a way that the trap disappears with the particle. This lattice model was studied in detail by Lawler, Bramson and Griffeath, Ben Arous and Ramirez, Gravner and Quastel. As $t \to \infty$, $\varrho(t,\cdot\,)$ inflates and $V(t,\cdot\,)$ deflates on a ball with radius $R(t) \sim (\lambda t/\omega_d)^{1/d}$ centered at the origin. We compute the asymptotics of $R(t)$ up to and including order 1, identify the shapes of $\varrho(t,\cdot\,)$ and $V(t,\cdot\,)$ near $R(t)$, as well as obtain their limiting profile away from $R(t)$ after appropriate scaling. Particular emphasis will be laid on the discussion of the most interesting two-dimensional scaling invariant case.