Abstract

We analyse a model of exhaustion of shared resources where allocation and deallocation requests are modelled by dynamic random variables as follows: Let $(E, \mathcal{A}, \mu, T)$ be a dynamic system where $(E,$ $mathcal A, \mu)$ is a probability space and T is a transformation defined on E. Let $d \geq 1$ and $f_1, \ldots, f_d$ be functions defined on E with values in $[0, \frac{1}{d}]$. Let $(X_i)_{i \geq 1}$ be a sequence of independent random vectors with values in $\mathbb{Z}^d$. Let $x \in E$ and $(e_j)_{1 \leq j \leq d}$ be the unit coordinate vectors of $\mathbb{Z}^d$. For every $i \geq 1$, the law of the random vector X_i is given by

$$\mathbb{P}(X_i = z) = \begin{cases} f_j(T^i x) & \text{if } \; z = e_j \\ \frac{1}{d} - f_j(T^i x) & \text{if } \; z = -e_j \\ \frac{1}{d} - f_j(T^i x) & \text{if } \; z = -e_j \\ 0 & \text{otherwise} \end{cases}$$

We write

$$S_0 = 0, \quad S_n = \sum_{i=1}^{n} X_i \text{ for } n \geq 1$$

for the $\mathbb{Z}^d$-random walk generated by the family $(X_i)_{i \geq 1}$. When T is a rotation on the torus then explicit calculations are possible. This stochastic model is a (small) step towards the analysis of distributed algorithms when allocation and deallocation requests are time dependent. It subsumes the models of colliding stacks and of exhaustion of shared memory considered in Louchard, Knuth, Maier and Yao. The technique is applicable to other stochastically modelled resource allocation protocols such as option pricing in financial markets and dam management problems.