Performance analysis of zone picking systems

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Zone-picking systems

- Popular order-picking system
- Storage area divided in order-picking zones
- Reduction of walking distances and congestion in aisles
- Flexible capacity and high-throughput ability
- Fit-for-use for a wide range of products and order profiles
Motivation

- Develop **fast and accurate** method to predict performance:
  - utilization of pick stations
  - system throughput
  - order lead time

- Method to **support design decisions**:
  - layout of the network
  - size of zones
  - location of items
  - number of pickers and zones
  - WIP level
Zone-picking systems
Zone-picking systems
Zone-picking systems
Zone-picking systems

Zone 1
Zone 2
Zone 3
Zone 4
Zone 5
Zone 6
Zone 7
Zone 8
Segment 1
Segment 2
Zone-picking systems

Pick-by-voice

Pick-by-light
Zone-picking systems
Zone-picking systems
Zone-picking systems
Zone-picking systems

/department of mechanical engineering
Zone-picking systems
Zone-picking systems

Disadvantage:

- congestion and blocking under heavy use
- leads to recirculation and long order lead times

Modeling:

- blocking is crucial aspect!
- describe elements (transport, zones) as network of queues

Method of analysis: queueing theory

Needed?
Pickers are equally fast, 10 circulating totes

**Question:** Replace one picker by a picker that is twice as fast. How does this affect mean order lead time? Throughput?

**Question:** Does your answer change in case of more totes? Less totes?
Multiple pickers or a single one?

4 pickers, or one picker that is four times faster?

Question: What do you prefer, 4 pickers or one fast picker?

Question: What do you prefer if pick time variability is high?

Question: What do you prefer if the load is low?
Multiple pickers or a single one?

4 pickers, or one picker that is four times faster?

The mean order lead time can be predicted by...

\[ E(S) \approx \frac{\Pi W}{1 - \rho} \frac{E(R)}{c} + E(B) \]
Layout of single-segment

Storage
Buffer
Zone

Weight check

System entrance/exit

Recirculation

Tote

Order picker
Modeling of single-segment

- \( N \) is number of totes
- \( M \) is number of zones
- \( \mathcal{S} \) is set of nodes; three types
  1. Entrance/exit, \( e \)
  2. Zones, \( \mathcal{Z} = \{z_1, \ldots, z_M\} \)
  3. Conveyors, \( \mathcal{C} = \{c_1, \ldots, c_{M+1}\} \)
- Each tote has class \( r \subseteq \mathcal{Z} \) of zones to be visited, for example, \( r = \{z_2, z_3\} \)
Closed queueing network with $\mathcal{C} = \{c_1, c_2, c_3\}$ and $\mathcal{Z} = \{z_1, z_2\}$
• **Entrance node** releases new totes one-by-one of class $r$ with probability $\psi_r$ at exponential rate $\mu_e$

• **Conveyor nodes** are delay nodes with a fixed delay of rate $\mu_i$

• **Zones** have:
  
  – $d_i \geq 1$ order pickers
  
  – Exponential pick times with rate $\mu_i$
  
  – Finite buffers of size $q_i$
Analysis of single-segment

- Distribution of network is **intractable**: Approximate!
- tote jumps over full zone and proceeds **as if** zone has been visited...
- Jump-over network has product-form solution!

- Flows of jump-over network should match with block-and-recirculate:

  passing tote is labeled $z_i$ **not visited** with probability $b_{z_i}$ and labeled $z_i$ **visited** otherwise, independent of whether the tote visited $z_i$ or not

- $b_{z_i}$ is blocking probability in block-and-recirculate network: Unknown!
Theorem:
Jump-over network has product-vorm stationary distribution:

\[
\pi(\bar{x}) = \frac{1}{G} \prod_{i \in S} \left( \frac{V_i}{\mu_i} \right)^{\bar{x}_i} \prod_{i \in C} \frac{1}{\bar{x}_i!} \prod_{i \in Z} \frac{1}{\gamma_i(\bar{x}_i)}
\]

where
- \(\bar{x}_i\) number of totes in node \(i\)
- \(G\) is normalizing constant
- \(V_i\) visiting frequency to node \(i\)
- \(\gamma_i\) is (queue dependent) service rate multiplier

Hence:
Jump-over network can be exactly evaluated by Mean Value Analysis (MVA)
Arrival theorem for closed queueing network:

Blocking probability $b_{zi}$ of zone $z_i$ is equal to:

$$b_{zi} = \pi_{zi} \left( d_{zi} + q_{zi} |N - 1\right),$$

where $\pi_{zi}(k|N)$ probability of $k$ totes in zone $z_i$ in network with $N$ totes

Remark:
Probabilities $\pi_{zi}(k|N)$ can be calculated recursively (over $N$) by MVA
Analysis of single-segment

- **Step 0:**
  Initialize \( b_{zi}^{(0)} = 0 \) and \( j = 0 \)

- **Step 1:**
  Calculate by means of MVA:
  1. Mean order lead times and throughput
  2. Distribution of totes per node

- **Step 2:**
  \( j = j + 1 \) and estimate new blocking probabilities
  \[ b_{zi}^{(j)} = \pi_{zi} (d_{zi} + q_{zi} | N - 1) \]

- **Step 3:**
  Return to Step 1 until \( |b_{zi}^{(j)} - b_{zi}^{(j-1)}| < \epsilon \)
## Results for single-segment

### Parameters single-segment test set (9600 cases)

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of zones</td>
<td>1,2,3,4,5,6,7,8</td>
</tr>
<tr>
<td>Number of totes</td>
<td>10,20,30,40,50,60,70,80</td>
</tr>
<tr>
<td>Transport mean of conveyors</td>
<td>20,30,40,50,60</td>
</tr>
<tr>
<td>Service mean of zones</td>
<td>10,15,20,25,30</td>
</tr>
<tr>
<td>Buffer size of zones</td>
<td>0,1</td>
</tr>
<tr>
<td>Number of order pickers</td>
<td>1,2,3</td>
</tr>
</tbody>
</table>
## Results for single-segment

<table>
<thead>
<tr>
<th>Zones</th>
<th>Error (%) in system throughput</th>
<th>Error (%) in circulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg.</td>
<td>SD.</td>
</tr>
<tr>
<td>1</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>2</td>
<td>0.67</td>
<td>0.84</td>
</tr>
<tr>
<td>3</td>
<td>0.78</td>
<td>1.03</td>
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<tr>
<td>4</td>
<td>0.73</td>
<td>1.05</td>
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<tr>
<td>5</td>
<td>0.64</td>
<td>1.00</td>
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<tr>
<td>6</td>
<td>0.54</td>
<td>0.91</td>
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<tr>
<td>7</td>
<td>0.45</td>
<td>0.81</td>
</tr>
<tr>
<td>8</td>
<td>0.38</td>
<td>0.71</td>
</tr>
</tbody>
</table>
Layout of multi-segment
Modeling of multi-segment

- $K$ is number of segments
- $N$ is number of totes
- $N^k$ is maximum number of totes in segment $k$
- $M$ is number of zones
- $\mathcal{S}$ is set of nodes; three types
  1. Entrance/exit nodes, $\mathcal{E} = \{e_0, e_1, \ldots, e_K\}$
  2. Zones, $\mathcal{Z} = \bigcup_{k=1}^{K} \mathcal{Z}^k$, $\mathcal{Z}^k = \{z^k_1, \ldots, z^k_{m^k}\}$
  3. Conveyors, $\mathcal{C} = \bigcup_{k=0}^{K} \mathcal{C}^k$, $\mathcal{C}^0 = \{c^0_1, \ldots, c^0_{K+1}\}$ and $\mathcal{C}^k = \{c^k_1, \ldots, c^k_{m^k+1}\}$
- Each tote has class $r \subseteq \mathcal{Z}$ of zones to be visited, for example, $r = \{z^1_2, z^2_3\}$
Corresponding closed queueing network
Analysis of multi-segment

- Blocking at two levels:
  1. when zone is full
  2. when segment is full

- Jump-over network:

  passing tote is labeled segment $k$ not visited with prob $B_k$, and labeled segment $k$ visited, independent of whether tote visited segment $k$ or not

- $B_k$ is blocking probability of segment $k$ in block-and-recirculate network: Unknown!
Analysis of multi-segment

Aggregation:

Replace segments by flow equivalent servers, with rates

\[ \mu_{FES_k}(n) = X^k(n), \quad n = 1, \ldots, N^k, \quad k = 1, \ldots, K \]

where \( X^k(n) \) is throughput of segment \( k \) in isolation
Aggregation of multi-segment

Segments replaced by flow equivalent servers
Aggregation van multi-segment

- Norton’s theorem:
  Aggregate network has same performance as jump-over network

- Analysis of aggregate network same as one of single-segment network!

- Arrival theorem:
  Blockings probability $B_k$ of segment $k$ is equal to:

  $$B_k = \prod_k (N^k|N - 1),$$

  where $\prod_k(n|N)$ is probability of $n$ totes in segment $k$ in network with $N$ totes
Analysis of multi-segment

- **Step 0:**
  Initialize \( b_{z_i}^{(0)} = B_k^{(0)} = 0 \) and \( j = 0 \)

- **Step 1:**
  Calculate for aggregate network and each segment by MVA:
  1. Mean order lead times and throughput
  2. Distribution of totes per node

- **Step 2:**
  \( j = j + 1 \) and estimate new blocking probabilities
  \[
  b_{z_i}^{(j)} = \pi_{z_i} \left( d_{z_i} + q_{z_i} | N - 1, N^k - 1 \right), \quad B_k^{(j)} = \Pi_k (N^k | N - 1)
  \]

- **Step 3:**
  Return to Step 1 until \( \left| b_{z_i}^{(j)} - b_{z_i}^{(j-1)} \right| < \epsilon \) and \( \left| B_k^{(j)} - B_k^{(j-1)} \right| < \epsilon \)
Results for multi-segment

Example using real-life data of large Dutch wholesaler of non-food:

- 4 segments
- 3 pick-by-light segments with each $2 \times 4$ zones
- 1 pallet pick with 3 zones
- Total:
  1. 24 pick-by-light zones
  2. 3 pallet pick zones

Compare 3 storage strategies:

1. Minimize expected number of segments to be visited (Current).
2. Balance work-load over segments (Balanced).
3. Random storage (Random).
Results for multi-segment

\[ X(N) = (h-1) \]
Conclusions

- Zone-picking system can be described by:
  
  closed multi-class queueing network with block-and-recirculate blocking

- Network can be approximated by jump-over network

- Excellent and fast estimates of performance by MVA and Norton