Understanding and Preventing Tacit Collusion among Telecommunication Operators

Patrick Maillé Maurizio Naldi Bruno Tuffin

Telecom Bretagne - Università di Roma “Tor Vergata” - INRIA

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Demand model

Demand for the two providers

$D_1 = D_{0,1} - b_1 p_1 + \beta_1 p_2$

$D_2 = D_{0,2} - b_2 p_2 + \beta_2 p_1$

- $D_{0,i}$ = Demand level if prices were set to zero
- $b_i > 0$ models negative effect of provider *i*'s price on his demand
- $\beta_i > 0$ models the positive effect of the concurrent's price

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Conditions on demand coefficients

1. A uniform price increase by any firm cannot produce a total demand increase $\Longrightarrow b_1 \geq \beta_2$ and $b_2 \geq \beta_1$
2. The direct effect of price b_i on demand D_i to be strictly larger than the indirect effect of the competitor's price β_i

$$\forall i \neq j, \quad \begin{cases} b_i > \beta_i \\ b_i \geq \beta_j. \end{cases}$$

The single shot game

Utilities

$$U_1(p_1, p_2) = p_1 D_1 = p_1(D_{0,1} - b_1 p_1 + \beta_1 p_2) \text{ (Provider 1)}$$
$$U_2(p_1, p_2) = p_2 D_2 = p_2(D_{0,2} - b_2 p_2 + \beta_2 p_1) \text{ (Provider 2)}$$

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Nash equilibrium under competition (each provider maximizing its utility)

$$(p_1^N, p_2^N) = \left(\frac{2b_2 D_{0,1} + \beta_1 D_{0,2}}{4b_1 b_2 - \beta_1 \beta_2}, \frac{2b_1 D_{0,2} + \beta_2 D_{0,1}}{4b_1 b_2 - \beta_1 \beta_2}\right)$$

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Nash equilibrium under collusion (maximization of joint utility)

$$p_1^* = \frac{2b_2 D_{0,1} + (\beta_1 + \beta_2) D_{0,2}}{4b_1 b_2 - (\beta_1 + \beta_2)^2}$$
$$p_2^* = \frac{2b_1 D_{0,2} + (\beta_1 + \beta_2) D_{0,1}}{4b_1 b_2 - (\beta_1 + \beta_2)^2}$$

The symmetric single shot game

Symmetry conditions

$$D_{0,1} = D_{0,2} = D_0, \qquad b_1 = b_2 = b, \qquad \beta_1 = \beta_2 = \beta$$

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Competition

$$p_1^N = p_2^N = \frac{D_0}{2b-\beta} := p^N$$

$$U_1^N = U_2^N = b\left(\frac{D_0}{2b-\beta}\right)^2 := U^N$$

Collusion

$$p_1^* = p_2^* = \frac{D_0}{2b-2\beta} := p^*$$

$$U_1^* = U_2^* = \frac{D_0^2}{4(b-\beta)}$$

Normalization of game results in the symmetric case (1/2)

	p^N	p^*
p^N	$(0,0)$	$\left(\frac{\beta^2}{2b(b-\beta)}, -\frac{\beta^2}{(2b-2\beta)^2}\right)$
p^*	$\left(-\frac{\beta^2}{(2b-2\beta)^2}, \frac{\beta^2}{2b(b-\beta)}\right)$	$\left(\frac{\beta^2}{4b(b-\beta)}, \frac{\beta^2}{4b(b-\beta)}\right)$

Providers' utilities depend on the price sensitivities b and β, but not on the demand D_0

Normalization of game results in the symmetric case (2/2)

Through normalization with $\frac{\beta^2}{(2b-2\beta)^2} = (1+\alpha)\frac{\beta^2}{4b(b-\beta)}$

	p^N	p^*
p^N	$(0,0)$	$(2,-1-\alpha)$
p^*	$(-1-\alpha,2)$	$(1,1)$

The repeated game

- The game is played repeatedly, each time a user chooses its provider
- Repeated playing provides room for collusion — Fisher 1989
- We need a utility measure over an extended time period

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For the provider i we have

$$V_i = (1-\delta)\sum_{t=0}^{+\infty} \delta^t U_i(s_t)$$

where $0 < \delta \leq 1$ = Discount factor

Equilibrium in the repeated game

If the discount factor δ is sufficiently close to 1
THEN
Providers playing joint-maximization prices at each period
is a
subgame-perfect Nash outcome of the repeated game

$$\Downarrow$$

We need to prevent tacit collusion (through regulatory tools)

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We need to prevent tacit collusion (through regulatory tools)
By reducing the discount factor

Price stability as a competition incentive

Providers are allowed to modify their prices every k periods

$$a_{km+\ell} = a_{km} \quad \forall m \in \mathbb{N}, \ell < k$$

The extended-time utility becomes

$$\begin{aligned}
V_i &= (1-\delta)\sum_{t=0}^{\infty}\delta^t U_i(a_t) \\
&= (1-\delta)\sum_{m=0}^{\infty}\delta^{km}\sum_{\ell=0}^{k-1}\delta^{\ell} U_i(a_{km+\ell}) \\
&= (1-\delta)\sum_{\ell=0}^{k-1}\delta^{\ell}\sum_{m=0}^{\infty}\delta^{km} U_i(a_{km}) \\
&= (1-\delta^k)\sum_{m=0}^{\infty}(\delta^k)^m U_i(a_{km})
\end{aligned}$$

A joint-maximization perfect strategy

	p^N	p^*
p^N	$(0, 0)$	$(2, -1-\alpha)$
p^*	$(-1-\alpha, 2)$	$(1, 1)$

The strategy

1. Play p^* while both do
2. If one provider deviates, then play p^N forever

is a perfect Nash equilibrium of the repeated game if the present value of future sanctions is larger than the immediate gain, i.e., the discount factor is $\delta^k > 1/2$

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For practical values of δ the stability period may be too long (e.g., 15 years when $\delta = 0.95$)

Joint maximization is not always beneficial

IF

1. $(D_{0,1}, D_{0,2}) = (1, 2)$ (a provider has a larger service basin)
2. $b_i = 2\beta_i = \gamma D_{0,i}$ (the price effects are proportional to the service basin)

THEN

the weaker provider would prefer the competitive strategy

	p_2^N	p_2^*
p_1^N	$(0.44, 0.88)$	$(0.54, 0.72)$
p_1^*	$(0.14, 1.25)$	$(0.38, 1.25)$

An example of success for the price stability tool

IF

1. $(D_{0,1}, D_{0,2}) = (3, 4)$ (a provider has a larger service basin)
2. $b_i = 2\beta_i = \gamma D_{0,i}$ (the price effects are proportional to the service basin)

THEN

the collusion gain is very small for provider 1

	p_2^N	p_2^*
p_1^N	$(1.33, 1.78)$	$(1.63, 1.42)$
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The collusion can be sustained if $\delta^k \geq 0.9845$
For a monthly interest rate of 0.5% that leads to a stability period slightly longer than 3 months

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- The competition between providers has been modelled as a repeated noncooperative game
- the repetition of the game allows providers to build credible threats to each other so as to maintain high prices, even if each provider could improve its short-term revenue by a price decrease
- Price stability could deter collusion though the stability period may be too long in the symmetric case