Characterizing Heavy-Tailed Distributions Induced by Retransmissions

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Abstract

Consider a generic data unit of random size L that needs to be transmitted over a channel of unit capacity. The channel availability dynamics is modeled as an i.i.d. sequence $\{A, A_i\}_{i\geqslant 1}$ that is independent of L. During each period of time that the channel becomes available, say A_i, we attempt to transmit the data unit. If $L \leq A_i$, the transmission is considered successful; otherwise, we wait for the next available period A_{i+1} and attempt to retransmit the data from the beginning. We study the asymptotic properties of the number of retransmissions N and the total transmission time T until the data is successfully transmitted. In the context of studying the completion times in systems with failures where jobs restart from the beginning, it was first recognized by Fiorini et al. (2005) and sheahan et al. (2006) that this model results in power law and, in general, heavy-tailed delays. The main objective of this paper is to uncover the detailed structure of this class of heavy-tailed distributions induced by retransmissions.

More precisely, we study how the functional dependence $(\mathbb{P}[L > x])^{-1} \approx \Phi((\mathbb{P}[A > x])^{-1})$ impacts the distributions of N and T; the approximation $\approx$ will be appropriately defined in the paper depending on the context. In the functional space of $\Phi(\cdot)$, we discover several functional criticality points that separate classes of different functional behavior of the distribution of N. For example, we show that if $\log(\Phi(n))$ is slowly varying, then $\log(\mathbb{P}[N > n])$ is essentially slowly varying as well. Interestingly, if $\log(\Phi(n))$ grows slower than $e^{\sqrt{\log n}}$ then we have the asymptotic equivalence $\log(\mathbb{P}[N > n]) \approx -\log(\Phi(n))$. However, if $\log(\Phi(n))$ grows faster than $e^{\sqrt{\log n}}$, this asymptotic equivalence does not hold and admits a different functional form. Similarly, different types of functional behavior are shown for moderately heavy tails (Weibull distributions) where $\log(\mathbb{P}[N > n]) \approx -(\log \Phi(n))^{1/(\beta+1)}$ assuming $\log \Phi(n) \approx n^{\beta}$, as well as the nearly exponential ones of the form $\log(\mathbb{P}[N > n]) \approx -n/(\log n)^{1/\gamma}, \gamma > 0$ when $\Phi(\cdot)$ grows faster than two exponential scales $\log \log (\Phi(n)) \approx n^{\gamma}$.

We also discuss the engineering implications of our results on communication networks since retransmission strategy is a fundamental component of the existing network protocols on all communication layers, from the physical to the application one.

Keywords: Retransmissions, Channel (systems) with failures, Restarts, Origins of heavy-tails (subexponentiality), Gaussian distributions, Exponential distributions, Weibull distributions, Log-normal distributions, Power laws.

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