Generalized Integrated Telegrapher Process and The Distribution of Related Stopping Times

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Abstract

Let $\{X(t), V(t), t \geq 0\}$ be a telegrapher process, with $V(0+) = 1$. The distribution of $X(t)$ is derived for the general case of an alternating renewal process, describing the length of time a particle is moving to the right or to the left. The distributions of the first crossing times of given levels $x = a$ and $x = -a$ are studied for M/G and for G/M processes.

Key Words: Telegrapher process, alternating renewal process, compound Poisson, upper and lower crossing times.

AMSC Subject Classification: 60G51, 60K15, 60K40

1 Introduction

The telegrapher's random process describes the motion of a particle on the real line, traveling at constant speed, whose direction is reversed at random times, according to the arrival epochs of a Poisson counting process $\{N(t), t \geq 0\}$. If v is the constant velocity of the particle, and if we assume that at time $t = 0$ the particle is at the origin and moves in a positive direction, then the position of the particle at time t is

$$X(t) = v \int_0^t (-1)^{N(s)} ds. \tag{1.1}$$

We assume here that $N(0) = 0$. Notice that in this simple model, the length of times at which the particle is traveling in the positive or negative direction are i.i.d. exponential

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